Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 13 May 2015 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

In triangle ABC, $\mathrm{AB} = 5\,\mathrm{cm}$, $\mathrm{BC} = 12\,\mathrm{cm}$ and $\mathrm{A\hat{B}C} = 100^\circ$.

(a) Find the area of the triangle. [2]

(b) Find AC. [2]

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2. [Maximum mark: 6]

From a group of five males and six females, four people are chosen.

(a) Determine how many possible groups can be chosen. [2]

(b) Determine how many groups can be formed consisting of two males and two females. [2]

(c) Determine how many groups can be formed consisting of at least one female. [2]

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3. [Maximum mark: 5]

(a) Sketch the graph of $y = (x-5)^2 - 2|x-5| - 9$, for $0 \le x \le 10$. [3]

(b) Hence, or otherwise, solve the equation $(x-5)^2 - 2|x-5| - 9 = 0$. [2]

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4. [Maximum mark: 7]

Emma acquires a new cell phone for her birthday and receives texts from her friends. It is assumed that the daily number of texts Emma receives follows a Poisson distribution with mean $m = 5$.

(a) (i) Find the probability that on a certain day Emma receives more than 7 texts.

(ii) Determine the expected number of days in a week on which Emma receives more than 7 texts. [4]

(b) Find the probability that Emma receives fewer than 30 texts during a week. [3]

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5. [Maximum mark: 7]

Consider the vectors given by $\boldsymbol{u} = \boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{v} = a\boldsymbol{i} + b\boldsymbol{j}$, where a and b are constants.

It is given that $\boldsymbol{u} \times \boldsymbol{v} = 4\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$, where c is a constant.

(a) Find the value of each of the constants a, b and c. [5]

(b) Hence find the Cartesian equation of the plane containing the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ and passing through the point $(0, 0, 0)$. [2]

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6. [Maximum mark: 6]

The graph of $y = \ln(5x + 10)$ is obtained from the graph of $y = \ln x$ by a translation of a units in the direction of the x-axis followed by a translation of b units in the direction of the y-axis.

(a) Find the value of a and the value of b. [4]

(b) The region bounded by the graph of $y = \ln(5x + 10)$, the x-axis and the lines $x = \mathrm{e}$ and $x = 2\mathrm{e}$, is rotated through 2π radians about the x-axis. Find the volume generated. [2]

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7. [Maximum mark: 9]

Consider the following system of equations

$$\begin{aligned} 2x + y + 6z &= 0 \\ 4x + 3y + 14z &= 4 \\ 2x - 2y + (\alpha - 2)z &= \beta - 12 \,. \end{aligned}$$

(a) Find conditions on α and β for which

(i) the system has no solutions;

(ii) the system has only one solution;

(iii) the system has an infinite number of solutions. [6]

(b) In the case where the number of solutions is infinite, find the general solution of the system of equations in Cartesian form. [3]

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(Question 7 continued)

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8. [Maximum mark: 10]

Farmer Bill owns a rectangular field, $10\,\text{m}$ by $4\,\text{m}$. Bill attaches a rope to a wooden post at one corner of his field, and attaches the other end to his goat Gruff.

(a) Given that the rope is $5\,\text{m}$ long, calculate the percentage of Bill's field that Gruff is able to graze. Give your answer correct to the nearest integer. [4]

(b) Bill replaces Gruff's rope with another, this time of length a, $4 < a < 10$, so that Gruff can now graze exactly one half of Bill's field.

Show that a satisfies the equation

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40\,.$$ [4]

(c) Find the value of a. [2]

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9. [Maximum mark: 6]

Natasha lives in Chicago and has relatives in Nashville and St. Louis.
Each time she visits her relatives, she either flies or drives.

When travelling to Nashville, the probability that she drives is $\frac{4}{5}$, and when travelling to St. Louis, the probability that she flies is $\frac{1}{3}$.

Given that the probability that she drives when visiting her relatives is $\frac{13}{18}$, find the probability that for a particular trip,

(a) she travels to Nashville; [3]

(b) she is on her way to Nashville, given that she is flying. [3]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 12]

Farmer Suzie grows turnips and the weights of her turnips are normally distributed with a mean of $122\,\text{g}$ and standard deviation of $14.7\,\text{g}$.

(a) (i) Calculate the percentage of Suzie's turnips that weigh between $110\,\text{g}$ and $130\,\text{g}$.

(ii) Suzie has 100 turnips to take to market. Find the expected number weighing more than $130\,\text{g}$.

(iii) Find the probability that at least 30 of the 100 turnips weigh more than $130\,\text{g}$. [6]

Farmer Ray also grows turnips and the weights of his turnips are normally distributed with a mean of $144\,\text{g}$. Ray only takes to market turnips that weigh more than $130\,\text{g}$. Over a period of time, Ray finds he has to reject 1 in 15 turnips due to their being underweight.

(b) (i) Find the standard deviation of the weights of Ray's turnips.

(ii) Ray has 200 turnips to take to market. Find the expected number weighing more than $150\,\text{g}$. [6]

11. [Maximum mark: 15]

A curve is defined by $x^2 - 5xy + y^2 = 7$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5y - 2x}{2y - 5x}$. [3]

(b) Find the equation of the normal to the curve at the point $(6, 1)$. [4]

(c) Find the distance between the two points on the curve where each tangent is parallel to the line $y = x$. [8]

Do **not** write solutions on this page.

12. [Maximum mark: 15]

A particle moves in a straight line, its velocity $v \text{ ms}^{-1}$ at time t seconds is given by $v = 9t - 3t^2$, $0 \leq t \leq 5$.

At time $t = 0$, the displacement s of the particle from an origin O is 3 m.

(a) Find the displacement of the particle when $t = 4$. [3]

(b) Sketch a displacement/time graph for the particle, $0 \leq t \leq 5$, showing clearly where the curve meets the axes and the coordinates of the points where the displacement takes greatest and least values. [5]

For $t > 5$, the displacement of the particle is given by $s = a + b\cos\frac{2\pi t}{5}$ such that s is continuous for all $t \geq 0$.

(c) Given further that $s = 16.5$ when $t = 7.5$, find the values of a and b. [3]

(d) Find the times t_1 and t_2 $(0 < t_1 < t_2 < 8)$ when the particle returns to its starting point. [4]

13. [Maximum mark: 18]

The equations of the lines L_1 and L_2 are

$$L_1 : \boldsymbol{r}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$L_2 : \boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}.$$

(a) Show that the lines L_1 and L_2 are skew. [4]

(b) Find the acute angle between the lines L_1 and L_2. [4]

(c) (i) Find a vector perpendicular to both lines.

(ii) Hence determine an equation of the line L_3 that is perpendicular to both L_1 and L_2 and intersects both lines. [10]

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